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LETTER TO THE EDITOR

The state with a spontaneous supercurrent on the surface of superfluid ^{3}He-B

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Abstract. A new metastable surface state on the boundary of ^{3}He-B is found analytically in the vicinity of the first-order transition line between ^{3}He-B and ^{3}He-A. This state consists of an A-phase layer separated from the B phase in bulk liquid by a conventional A–B interface. As distinct from all other surface states obtained by calculation so far, this state has a spontaneous mass supercurrent along the surface.

Several different experiments have indicated the possibility of there being a new state on the surface of superfluid ^{3}He–B. Fal'ko (1985), trying to explain the first-order phase transition observed in a gyroscopic experiment (Pekola and Simola 1985), proposed that in addition to the conventional surface layer with a planar state on the boundary of the vessel (Cross 1977) there may exist a surface state with spontaneously broken symmetry that has a spontaneous mass supercurrent along the boundary.

Thuneberg (1986), motivated by the same experiment as well as by an experiment performed by Ling *et al* (1984), calculated the surface layer structure within the Ginzburg–Landau theory near the normal–superfluid transition temperature T_c. He found that, in addition to the most symmetric conventional surface state, with surface energy $\sigma_B^{symm} = 0.76\xi(T)f_B$ (here $\xi(T)$ is the temperature-dependent coherence length and f_B is the superfluid condensation energy density in ^{3}He–B), at high pressure there exists a less symmetric metastable state together with the A-phase layer on the boundary. It has higher energy, $\simeq 1.20\xi(T)f_B$, and corresponds to the local minimum of the Ginzburg–Landau free energy. It becomes absolutely stable if a high enough external supercurrent is applied. Salomaa and Volovik (1989), who tried to explain an abnormal parity effect observed in NMR experiments on the vortex free state (Hakonen and Nummila 1987), proposed a phenomenological model for the boundary conditions that makes this state absolutely stable in some cases even without an external supercurrent. This is, however, not yet confirmed in the microscopic theory.

The metastable surface state found by Thuneberg is reminiscent, as regards its symmetry and order parameter structure, of the A–B walls trapped by the vessel boundary with the A-phase side of the wall attached to the boundary of the vessel. In the open geometry there are several types of A–B interfaces, differing in internal symmetry (Schopohl 1987, Salomaa 1988). The equilibrium A–B interface (Cross 1977, Kaul and Kleinert 1980), which has the energy $\sigma_{AB}^{(0)} \simeq 1.25\xi(T)f_B$, has the l-vector oriented in the plane of the interface. Also, the metastable A–B wall exists (Schopohl 1987, Salomaa 1988) with slightly higher energy $\sigma_{AB}^{(\pi/2)} \simeq 1.5\xi(T)f_B$ and with the l-vector

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oriented along the normal to the A–B wall. The A–B wall, which corresponds to the Thuneberg solution, has $\boldsymbol{l}$-oriented along the normal to the wall to satisfy the boundary condition on the boundary of the vessel. According to the classification scheme developed by Zhang *et al* (1987), the Thuneberg state belongs to symmetry class 11. This symmetry still contains the elements that prohibit the spontaneous supercurrent in the absence of an external mass current. Salomaa and Volovik (1989), within their model, found another surface state with the same symmetry class, 11. In this state the A phase is not fully developed near the boundary of the vessel.

Zhang *et al* (1987), who made a symmetry classification of the surface states, have calculated the surface layer structure within the quasiclassical theory for different roughnesses of the wall. They found another metastable state that has the same symmetry as the conventional one (the most symmetric class, 17, in their classification), and therefore also has no spontaneous supercurrent.

Here we show that the metastable surface state with a spontaneous supercurrent exists at least in the vicinity of the first-order A–B transition line $T_{AB}(P)$ on the T–P plane (P is the pressure). This surface state belongs to class 3 and also corresponds to the A-phase layer on the boundary separated from the bulk B phase by the A–B interface. However, as distinct from the Thuneberg state, this A–B wall is of the conventional type, with $\boldsymbol{l}$ parallel to the interface. Since at the boundary of the vessel $\boldsymbol{l}$ is normal to the wall and near the interface it is parallel to the wall, there is $\boldsymbol{l}$-texture in the A-phase layer between the boundary and the A–B wall. This texture on the one hand stabilizes this surface structure and on the other hand produces the net mass supercurrent along the surface, which comes from that part of the orbital current that is proportional to $C\nabla \times \boldsymbol{l}$.

We now consider this new surface state in the vicinity of the A–B transition line $T_{AB}(P)$ where the bulk energies F_B and F_A are close, and as a result the width of the A-phase layer near the boundary essentially exceeds the coherence length and the width of the A–B wall. In this case one may consider the A–B wall as a rigid A–B wall in bulk liquid, and for the texture in the A-phase layer one can use the London approximation instead of the Ginzburg–Landau theory, and therefore our considerations are valid not only at T_c but also far from T_c, i.e. near the $T_{AB}(P)$ line in its entirety. The energy of this surface structure as compared with the surface energy σ_B^{symm} of the conventional surface structure contains (i) the energy of the A–B wall, (ii) the surface energy of the A phase, which is less than σ_B^{symm} and equals zero for specular boundary conditions (Cross 1977), (iii) the bulk energy of the A-phase layer of thickness L, and (iv) the gradient energy of the $\boldsymbol{l}$-field in the A-phase layer:

$$\sigma_B^{new} - \sigma_B^{symm} = \sigma_{AB} + \sigma_A - \sigma_B^{symm} + (F_A - F_B)L + \int_0^L \mathrm{d}x\,[K_1(\nabla\cdot\boldsymbol{l})^2 + K_2(\boldsymbol{l}\cdot\nabla\times\boldsymbol{l})^2 + K_3|\boldsymbol{l}\times(\nabla\times\boldsymbol{l})|^2]. \tag{1}$$

Here the x axis is along the normal to the wall with $x = 0$ on the boundary of the vessel and $x = L$ on the A–B interface.

In the presence of an external superfluid velocity field $\boldsymbol{v}_s$ one should add (v) the superfluid kinetic energy of the A-phase layer compared with the superfluid kinetic energy of the B phase, and (vi) the interaction of the velocity field with the orbital current, $\boldsymbol{J}_{orb}$, produced by the $\boldsymbol{l}$-texture in the A-phase layer:

$$\Delta\sigma = \int_0^L \mathrm{d}x\,\tfrac{1}{2}[\rho_{sA}^{\parallel}(\boldsymbol{l}\cdot\boldsymbol{v}_s)^2 + \rho_{sA}^{\perp}|\boldsymbol{l}\times\boldsymbol{v}_s|^2 - \rho_{sB}v_s^2] + \boldsymbol{v}_s\cdot\boldsymbol{J}_{orb} \tag{2a}$$

$$J_{\text{orb}} = \int_0^L \mathrm{d}x \frac{\hbar}{M} [C \boldsymbol{v}_s \cdot \boldsymbol{\nabla} \times \boldsymbol{l} - C_0(\boldsymbol{l} \cdot \boldsymbol{v}_s)(\boldsymbol{l} \cdot \boldsymbol{\nabla} \times \boldsymbol{l})]. \tag{2b}$$

The gradient energy of the $\boldsymbol{l}$-field in the A-phase layer depends on the boundary condition for the $\boldsymbol{l}$-vector on the A–B wall at $x = L$. For the equilibrium A–B wall in bulk liquid the $\boldsymbol{l}$-vector is parallel to the wall; however, the texture in the A-phase layer tends to change this condition. Thus one must find the boundary condition in a self-consistent manner. We show here that in the vicinity of T_{AB} the boundary condition at the A–B wall ($\boldsymbol{l}$ is parallel to the wall) is not perturbed. We consider the planar texture in which $\boldsymbol{l}$ depends only on the x coordinate and is constrained in the (x, z) plane with z being arbitrarily chosen along the wall:

$$\boldsymbol{l} = (\cos \alpha(x), 0, \sin \alpha(x)). \tag{3}$$

We assume the following boundary conditions for $\boldsymbol{l}$: $\alpha(0) = 0$ and $\alpha(L) = \pi/2 - \delta$, where δ is the parameter of minimization which enters both the gradient energy and the energy of the A–B wall, $\sigma_{AB}(\delta)$. The latter is minimal when $\delta = 0$, and therefore for small δ one has

$$\sigma_{AB}(\delta) = \sigma_{AB}^{(0)} + D\delta^2 \tag{4}$$

where D may be estimated from $D \simeq \sigma_{AB}^{(\pi/2)} - \sigma_{AB}^{(0)}$. Below it will be shown that, in the vicinity of T_{AB}, the equilibrium $\delta \ll 1$ and may be neglected in calculating the structure of the $\boldsymbol{l}$-field.

In terms of $\alpha(x)$ the gradient energy of the $\boldsymbol{l}$-field is

$$F_{\text{grad}} = \int_0^L \mathrm{d}x\, (K_1 \sin^2 \alpha + K_3 \cos^2 \alpha)(\partial_x \alpha)^2. \tag{5}$$

Minimization of (5) with respect to $\alpha(x)$, taking into account the boundary conditions, gives the following results:

$$x(\alpha) = L\, \mathrm{E}(\alpha, k)/\mathrm{E}(\pi/2 - \delta, k) \tag{6}$$

$$F_{\text{grad}} = (\mathrm{E}(\pi/2 - \delta, k))^2\, K_3/L \tag{7}$$

where

$$\mathrm{E}(\alpha, k) = \int_0^\alpha \mathrm{d}u \sqrt{1 - k^2 \sin^2 u} \tag{8}$$

is the elliptical function and $k^2 = (K_3 - K_1)/K_3$. Further minimization of $F_{\text{grad}} + (F_A - F_B)L$ with respect to L gives for L and $F_{\text{grad}} + (F_A - F_B)L$ the following equations:

$$L = \sqrt{K_3/(F_A - F_B)}\mathrm{E}(\pi/2 - \delta, k) \tag{9}$$

$$F_{\text{grad}} + (F_A - F_B)L = 2\sqrt{K_3(F_A - F_B)}\mathrm{E}(\pi/2 - \delta, k). \tag{10}$$

Finally, one should minimize (10) together with the energy of the A–B wall in (4) with respect to δ. Note that near T_{AB}, where $(F_A - F_B) \ll F_B$, the size of the A-phase layer is large, $L \gg \xi$, which justifies the proposed approach. This also leads to the small value of δ. Since for small $\delta \ll 1$ one has $\mathrm{E}(\pi/2 - \delta, k) \simeq \mathrm{E}(\pi/2, k) - \delta\sqrt{K_1/K_3}$, the minimization with respect to δ gives

$$\delta = \sqrt{K_1(F_A - F_B)}/D. \tag{11}$$

This is small in the vicinity of T_{AB} which means that in this temperature region the A–B wall in the new surface state may be considered as rigid with the boundary condition $\delta = 0$ ($\boldsymbol{l}$ is parallel to the A–B wall) which is practically unaffected by the $\boldsymbol{l}$-texture in the A-phase layer. Thus the total energy of the new surface state in the leading approximation for the small parameter $(F_A - F_B)/F_B$ is

$$\sigma_B^{new} - \sigma_B^{symm} = \sigma_{AB}^{(0)} + (\sigma_A - \sigma_B^{symm}) + 2\sqrt{K_3(F_A - F_B)}\,\mathrm{E}(\pi/2, \sqrt{1 - K_1/K_3}). \quad (12)$$

We now consider the symmetry and spontaneous supercurrent in the new state.

The conventional surface state of class 17 is the most symmetric state with the following elements of symmetry: the continuous group of rotations $C_{\infty,x}$ about the $\hat{x}$ axis; time inversion symmetry T; and reflections R_y and R_z with respect to the planes (z, x) and (y, x) respectively. In the surface state of class 11 with the A-phase layer near the boundary of the vessel, which was found by Thuneberg (1986), this symmetry group H_{17} is broken into the subgroup H_{11} which contains the element $C_{2,x}$ (rotation by π about the x axis), and combined elements TR_y and TR_z. In our new metastable state the symmetry is broken further, with the only non-trivial element TR_y: $TR_y\, \boldsymbol{l}(x) = \boldsymbol{l}(x)$. This corresponds to class 3 according to the classification scheme given by Zhang *et al* (1987). The direction of the y axis is arbitrary which reflects the degeneracy of the state of class 3.

In the states of classes 17 and 11 the spontaneous supercurrent along the surface, J_z or J_y, is prohibited due to the symmetry element $C_{2,x}$ since $C_{2,x}J_{y,z} = -J_{y,z}$. This element is absent in the new state and the element TR_y of the new state allows the supercurrent along the y axis. This supercurrent is concentrated in the A-phase layer and comes from the orbital current $\boldsymbol{J}_{orb}$ of $\boldsymbol{l}$-texture. According to (2*b*) and (3) and the boundary conditions for the $\boldsymbol{l}$-vector one has

$$\boldsymbol{J}_{orb} = (\hbar/M)C\hat{\boldsymbol{y}}. \quad (13)$$

The direction of the spontaneous supercurrent may be fixed by the external supercurrent with the superfluid velocity $\boldsymbol{v}_s$. If the velocity $\boldsymbol{v}_s$ is small the leading interaction of $\boldsymbol{v}_s$ with $\boldsymbol{l}$ in (2) is linear in $\boldsymbol{v}_s$. This gives the orientation of the spontaneous supercurrent $\boldsymbol{J}_{orb}$ as opposite to $\boldsymbol{v}_s$. In this case the total energy of the new state under external superflow as compared with the conventional state is

$$\sigma_B^{new} - \sigma_B^{symm} = \sigma_{AB}^{(0)} + (\sigma_A - \sigma_B^{symm}) + 2\sqrt{K_3(F_A - F_B + \tfrac{1}{2}(\rho_{sA}^{\perp} - \rho_{sB})v_s^2)}$$
$$\times\, \mathrm{E}(\pi/2, \sqrt{1 - K_1/K_3}) - (\hbar/M)Cv_s. \quad (14)$$

In the above, we have found analytically a new surface state on the boundary of ^{3}He–B which corresponds to a local minimum of the free energy in the vicinity of the entire transition line T_{AB} between ^{3}He–B and ^{3}He–A. This surface state belongs to symmetry class 3 according to the classification scheme given by Zhang *et al* (1987) which allows the existence of the mass supercurrent along the surface. In the vicinity of T_{AB} the new state may be considered as an A-phase surface layer of large thickness, as compared with the coherence length, separated from the bulk B phase by a well defined A–B interface of conventional type. Far from T_{AB} the surface state cannot be separated into an A-phase layer and A–B wall, and our considerations cannot be applied here. However, this state has symmetry of class 3, which is different from that of the previously considered surface states of classes 17 and 11, and this symmetry cannot change continuously. Therefore, one should expect the state of symmetry class 3 to persist in a modified form well below T_{AB} until some critical temperature is reached at which it becomes unstable towards one of two other states. In principle, it is not impossible for the metastable state with a spontaneous supercurrent to become absolutely stable far from T_{AB}. Experimentally, the metastable state may be created via a continuous transition from ^{3}He–A to ^{3}He–B when the A–B wall moves slowly to the boundary of the vessel.

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